Chap 1 Fundamental Concepts

1.1 SI Units and Prefixes

SI Units

Quantity	Unit	Symbol	Derived unit
length	meter	m	
mass	kilogram	kg	
time	second	s	
current	ampere	A	
voltage	volt	V	$kg \cdot m^2 \cdot A^{-1} \cdot s^{-3}$
power	watt	W	$kg \cdot m^2 \cdot s^{-3}$
energy	joule	J	$kg \cdot m^2 \cdot s^{-2}$
charge	coulomb	C	$A \cdot s$
resistance	ohm	Ω	$kg \cdot m^2 \cdot A^{-2} \cdot s^{-3}$
conductance	siemens	S	$A^2 \cdot s^3 \cdot kg^{-1} \cdot m^{-2}$
capacitance	farad	F	$A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-2}$
inductance	henry	H	$kg \cdot m^2 \cdot A^{-2} \cdot s^{-2}$
requency	hertz	Hz	s^{-1}

* 方程式等號兩邊相加減之每一項的單位都必須相同。

* 電學單位可利用動能$\frac{1}{2}mv^2$與電能IVT、QV、$\frac{1}{2}CV^2$、$\frac{1}{2}LI^2$之單位轉換

SI Prefix	G	M	k	m	μ	n	p
10^{3n}	10^9	10^6	10^3	10^{-3}	10^{-6}	10^{-9}	10^{-12}

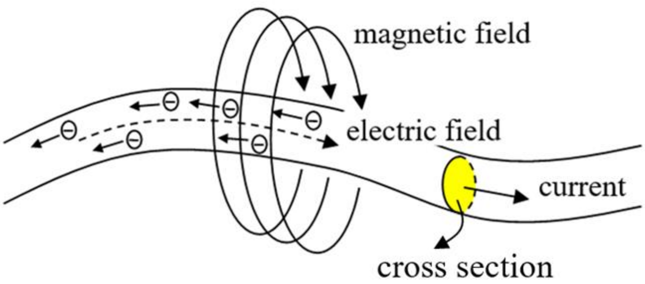
<u>Example</u>: $R = 22\times10^3\Omega = 22k\Omega$

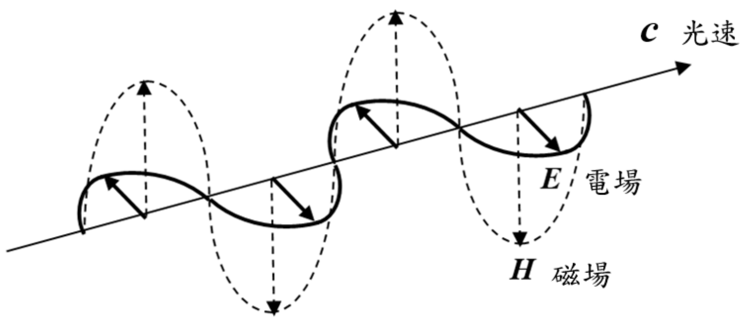
$L = 0.3\times10^{-3}H = 0.3mH$

$C = 4.7\times10^{-6}F = 4.7\mu F$

1.2 Field Sources and Fields

Field Sources		Electric field		Magnetic field	
		Static	Time-Varying	Static	Time-Varying
Charge	Stationary	■			
	Moving		■		
Current	Direct			■	
	Alternating				■
Electromagnetic wave			■		■





* 質量 m 之電荷 q 受力

重力 $\boldsymbol{F}_m = m\boldsymbol{g}$ 電力 $\boldsymbol{F}_E = q\boldsymbol{E}$ 磁力 $\boldsymbol{F}_B = q\boldsymbol{v} \times \boldsymbol{B}$

1.3 Electric Charges and Current

- The free charges in an electric wire are electrons, $e^- = -1.602\times 10^{-19}\,\mathrm{C}$.
- The electrons are driven to move by electric field and form a current.
- The current *I* through the cross section of an electric wire is defined as

  (1.3-1) $$I = \frac{\Delta q}{\Delta t} \quad [\mathrm{C/s}]$$

  where Δq is the charge through a cross section in a time period Δt.

- Instantaneous current

  (1.3-2) $$i(t) = \lim_{\Delta t\to 0}\frac{\Delta q}{\Delta t} = \left.\frac{q(t+dt)-q(t)}{dt}\right|_{dt\to 0} = \frac{dq(t)}{dt}$$

  On the other hand,

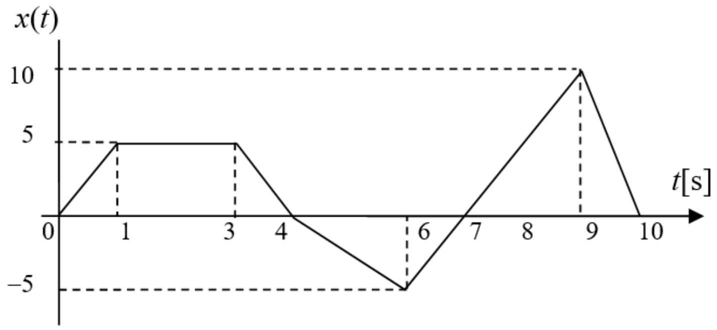
  (1.3-3) $$q(t) = \int_{-\infty}^{t} i(\tau)\,d\tau = q(t_0) + \int_{t_0}^{t} i(\tau)\,d\tau, \qquad q(t_0) = \int_{-\infty}^{t_0} i(\tau)\,d\tau$$

- Average current in $t\in[t_1, t_2]$

  (1.3-4) $$I_{avg} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i(\tau)\,d\tau$$

* 為何一打開電源開關燈泡瞬間就亮了？是電子在電線中快速移動嗎？
並非如此，主要是因為電源一開就隨即產生電場沿著電線以光速傳遞，此時電線中所有的電子幾乎同時受到電力的作用開始移動，電流彷彿瞬間形成以點亮燈泡，不過真正實景卻是電子移動速度相當緩慢，約 10^{-4} m/s。
[請嘗試以水龍頭一打開馬上就有自來水流出的情況來解釋。]

Example: (1) If $x(t) = q(t)$[μC], determine $i(t)$.
(2) If $x(t) = i(t)$[mA], determine $q(t)$.



1.4 Voltage and Power

- A charge q moving from P_1 to P_2 consumes electric work

  (1.4-1) $w = q(V_1 - V_2) = qv$ [J]

  where V_1 and V_2 are the potentials at P_1 and P_2 and $v = V_1 - V_2$ is the voltage between P_1 and P_2.

- Instantaneous power

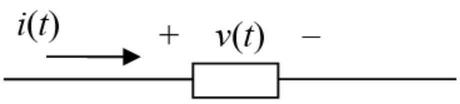
  (1.4-2) $p(t) = \dfrac{dw(t)}{dt} = \dfrac{dw(t)}{dq(t)}\dfrac{dq(t)}{dt} = v(t)i(t)$ [W, J/s]

  $$\Rightarrow \ w(t) = w(t_0) + \int_{t_0}^{t} p(\tau)\,d\tau$$

- Average power in $t \in [t_1, t_2]$

  (1.4-3) $P_{avg} = \dfrac{w(t_2) - w(t_1)}{t_2 - t_1} = \dfrac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(\tau)\,d\tau$

- Passive Sign Convention (PSC)



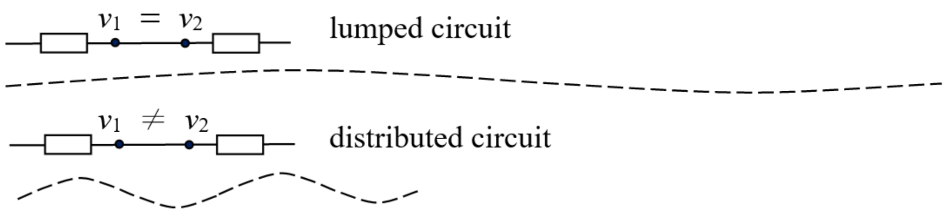
* 元件上電流 $i(t)$的方向與電壓 $v(t)$的正負端可以任意選定，
但是只有依據 PSC 所選定的 $i(t)$與 $v(t)$，
其乘積 $p(t)=v(t)i(t)$才是消耗或儲存在元件上功率。
在一般情況下，若未特別標明，元件的電壓與電流視為以 PSC 表示。

<u>Example</u>: Consider a component in an electric circuit. If $v(t) = 3\sin(\pi t/3)$ and $i(t) = 4\cos(\pi t/3)$, determine the instantaneous power $p(t)$ and the average power P_{avg} for $0 \le t \le 1$.

1.5 Kirchhoff's Laws

- KVL: Kirchhoff's voltage law
  KCL: Kirchhoff's current law

- A lumped circuit is constructed by lumped components with dimension much less than the wavelength of electric waves.



- A loop with *n* components in series

  KVL: $$\sum_{k=1}^{n} v_k = 0$$

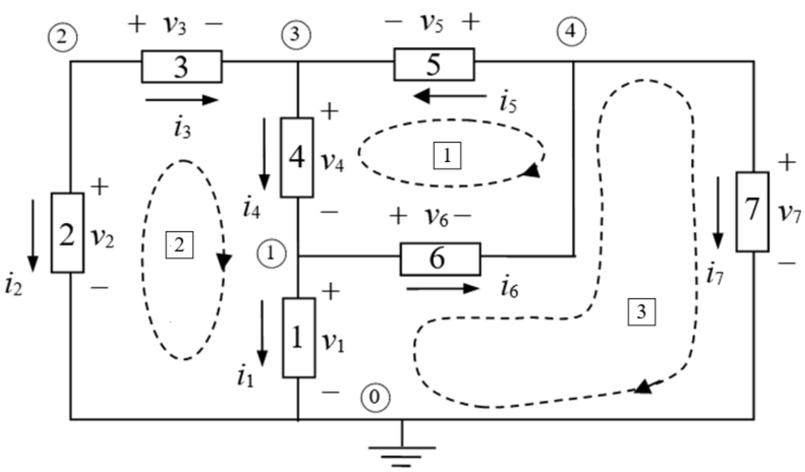
  where v_k is the voltage across the *k*-th component.

- A node with *m* components connecting to it

  KCL: $$\sum_{k=1}^{m} i_k = 0$$

  where i_k is the current through the *k*-th component away(or into) the node.

Example: Write KVL for the loops [1] to [3] and KCL for the nodes ① to ④.



1.6 Conservation of Energy

- The law of conservation of energy

(1.6-1) $$\sum_{k=1}^{n} w_k(t) = \sum_{k=1}^{n} \int_{-\infty}^{t} p_k(\tau)\, d\tau = \sum_{k=1}^{n} \int_{-\infty}^{t} v_k(\tau) i_k(\tau)\, d\tau = 0$$

which implies

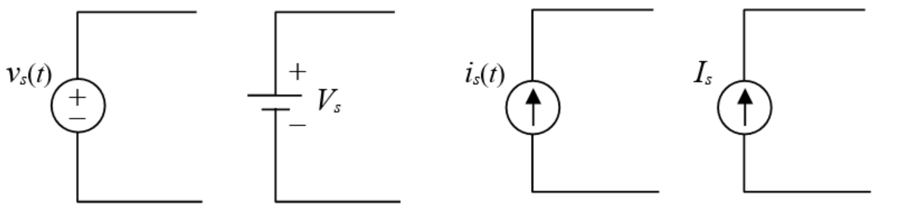
(1.6-2) $$\sum_{k=1}^{n} w_k(t) = \int_{-\infty}^{t} \left(\sum_{k=1}^{n} v_k(\tau) i_k(\tau) \right) d\tau = \int_{-\infty}^{t} \left(\sum_{k=1}^{n} p_k(\tau) \right) d\tau = 0 .$$

- The total power is also conserved

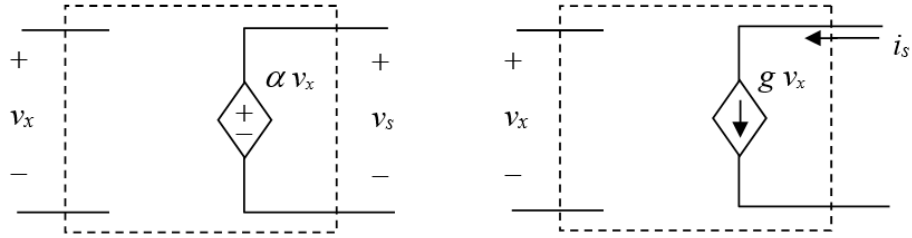
(1.6-3) $$\sum_{k=1}^{n} p_k(\tau) = \sum_{k=1}^{n} v_k(t) i_k(t) = 0$$

1.7 Ideal Sources

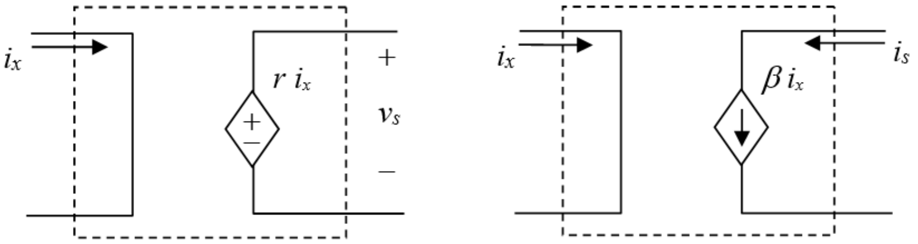
- Independent voltage/current sources



- Dependent Sources



VCVS VCCS



CCVS CCCS

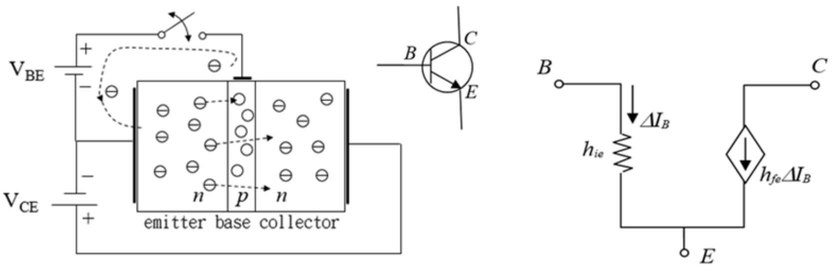
VCVS: Voltage-Controlled Voltage Source

VCCS: Voltage-Controlled Current Source

CCVS: Current-Controlled Voltage Source

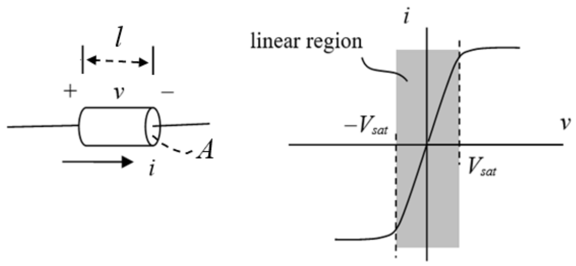
CCCS: Current-Controlled Current Source

<u>Example</u>: BJT Transistor, CCCS

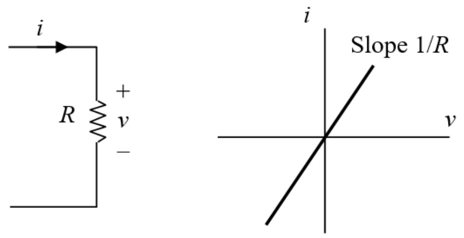


1.8 Resistors

- Nonlinear Resistor



- Linear Resistor



- Resistance

(1.8-1) $R = \rho \dfrac{l}{A}$ [Ω]

ρ : resistivity, A : cross area, l : length

- Ohm's Law

(1.8-2) $v(t) = R \cdot i(t)$ or $i(t) = G \cdot v(t)$

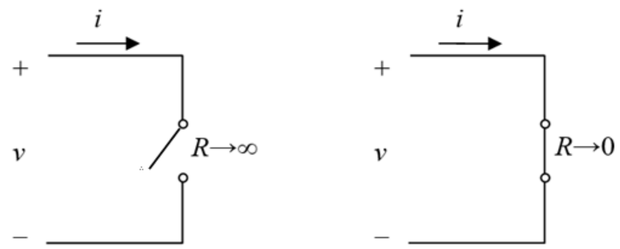
where $G = R^{-1}$ [S] is called the conductance.

- Power dissipated in resistor

(1.8-4) $p(t) = i(t)v(t) = Ri^2(t) = \dfrac{v^2(t)}{R}$ [W]

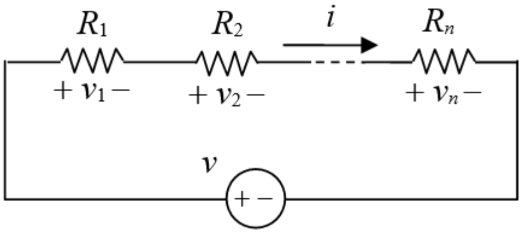
(1.8-5) $p(t) = i(t)v(t) = Gv^2(t) = \dfrac{i^2(t)}{G}$ [W]

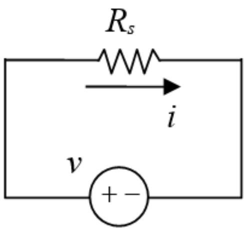
- Open circuit and Short circuit



1.9 Resistors in Series and parallel

1.9.1 Resistors in Series





- Mathematic model

(1.9.1-1) $i_1(t)=i_2(t)=\cdots=i_n(t)=i(t)$

(1.9.1-2) $v_k(t)=R_k i_k(t)=R_k i(t)$

KVL: $v(t)=v_1(t)+v_2(t)+\cdots+v_n(t)=(R_1+R_2+\cdots+R_n)i(t)=R_s i(t)$

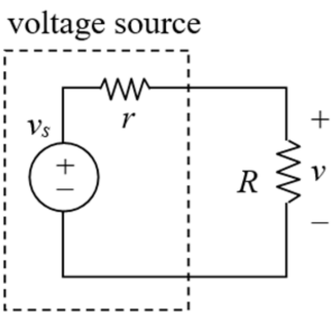
$$\Rightarrow\ i(t)=\frac{v(t)}{R_1+R_2+\cdots+R_n}=\frac{v(t)}{R_s}$$

$$\Rightarrow\ v_k(t)=R_k i(t)=\frac{R_k}{R_s}v(t)$$

- Equivalent resistance

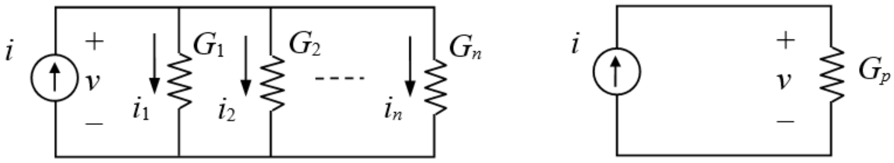
(1.9.1-3) $R_s=R_1+R_2+\cdots+R_n$

- Practical voltage source



(1.9.1-4) $v(t)=\dfrac{R}{R+r}v_s(t)$, $\quad r\ll R$

1.9.2 Resistors in Parallel



- Mathematic model

(1.9.2-1) $v_1(t) = v_2(t) = \cdots = v_n(t) = v(t)$

(1.9.2-2) $i_k(t) = G_k v_k(t) = G_k v(t)$

KCL: $i(t) = i_1(t) + i_2(t) + \cdots + i_n(t) = (G_1 + G_2 + \cdots + G_n)v(t) = G_p v(t)$

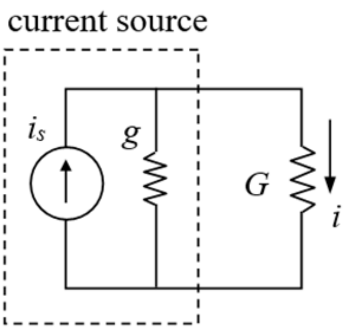
$$\Rightarrow \; v(t) = \frac{i(t)}{G_1 + G_2 + \cdots + G_n} = \frac{i(t)}{G_p}$$

$$\Rightarrow \; i_k(t) = G_k v(t) = \frac{G_k}{G_p} i(t)$$

- Equivalent conductance

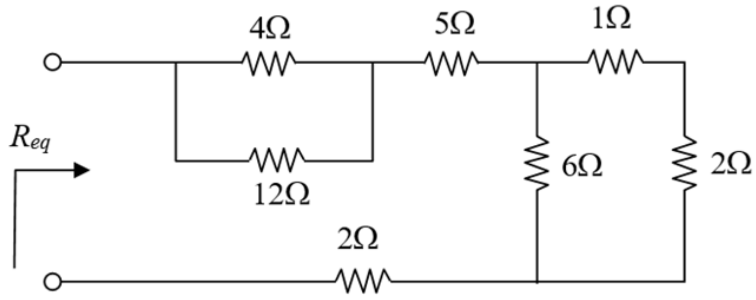
(1.9.2-3) $G_p = G_1 + G_2 + \cdots + G_n$ or $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}$

- Practical current source

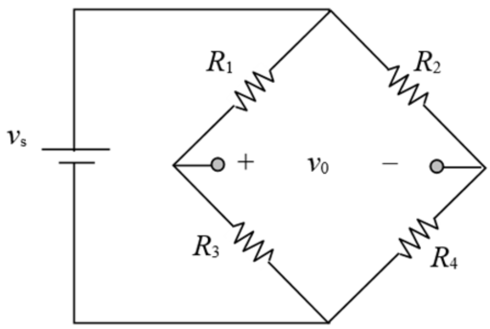


(1.9.2-4) $i(t) = \dfrac{G}{G+g} i_s(t), \qquad g \ll G$

<u>Example</u>: Determine the equivalent resistance R_{eq}

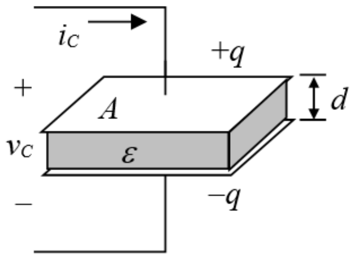


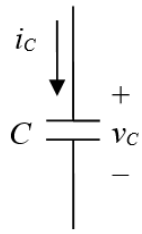
<u>Example</u>: Determine the voltage v_0.



1.10 Capacitors

- Linear capacitor





(1.10-1) $q(t) = Cv_C(t)$

- Capacitance

(1.10-2) $C = \varepsilon \dfrac{A}{d}$ [F, C/V]

ε : dielectric coefficient, A : plate area, d : distance

- **Component equations** (元件方程式)

(1.10-3) $i_C(t) = C\dfrac{dv_C(t)}{dt}$

(1.10-4) $v_C(t) = \dfrac{1}{C}\int_{-\infty}^{t} i_C(\tau)\,d\tau = v_C(t_0) + \dfrac{1}{C}\int_{t_0}^{t} i_C(\tau)\,d\tau$

where $v_C(t_0) = \dfrac{1}{C}\int_{-\infty}^{t_0} i_C(\tau)\,d\tau$.

- Continuity of capacitor's voltage

(1.10-5) $v_C(t^-) = v_C(t) = v_C(t^+)$

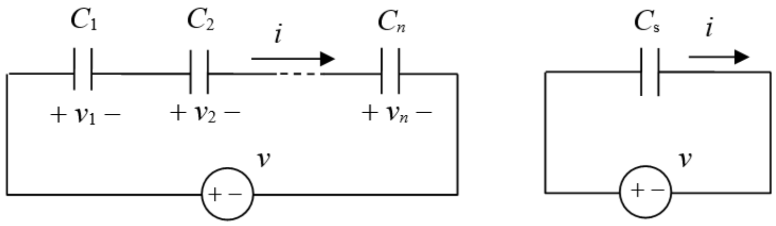
- Stored power and energy

(1.10-6) $p_C(t) = v_C(t)i_C(t) = v_C(t)\left(C\dfrac{dv_C(t)}{dt}\right) = \dfrac{d}{dt}\left(\dfrac{1}{2}Cv_C^2(t)\right)$

(1.10-7) $w_C(t) = \int_{t_0}^{t} p_C(\tau)\,d\tau = \dfrac{1}{2}Cv_C^2(\tau)\Big|_{v_C(t_0)}^{v_C(t)} = \dfrac{1}{2}Cv_C^2(t) - \dfrac{1}{2}Cv_C^2(t_0)$

1.11 Capacitors in Series and Parallel

1.11.1 Capacitors in Series



- Mathematic model

(1.11.1-1) $v(t) = v_1(t) + v_2(t) + \cdots + v_n(t) = \sum_{k=1}^{n} v_k(t)$

(1.11.1-2) $v_k(t) = \frac{1}{C_k}\int_{-\infty}^{t} i(\tau)\,d\tau = v_k(t_0) + \frac{1}{C_k}\int_{t_0}^{t} i(\tau)\,d\tau$

KVL: $v(t) = \sum_{k=1}^{n} \frac{1}{C_k}\int_{-\infty}^{t} i(\tau)\,d\tau$

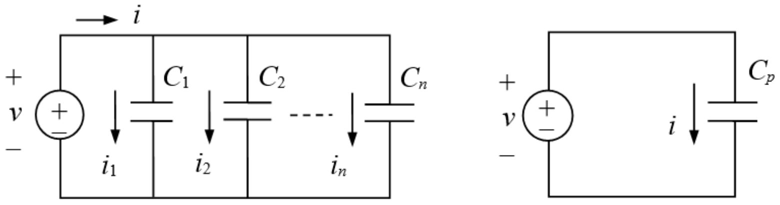
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)\int_{-\infty}^{t} i(\tau)\,d\tau = \frac{1}{C_s}\int_{-\infty}^{t} i(\tau)\,d\tau$$

- Equivalent capacitance

(1.11.1-3) $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

(1.11.1-4) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)^{-1}$

1.11.2 Capacitors in Parallel



- Mathematic model

(1.11.2-1) $i(t) = i_1(t) + i_2(t) + \cdots + i_n(t) = \sum_{k=1}^{n} i_k(t)$

(1.11.2-2) $i_k(t) = C_k \dfrac{dv(t)}{dt}$

KCL: $i(t) = \sum_{k=1}^{n} C_k \dfrac{dv(t)}{dt} = (C_1 + C_2 + \cdots + C_n)\dfrac{dv(t)}{dt} = C_p \dfrac{dv(t)}{dt}$

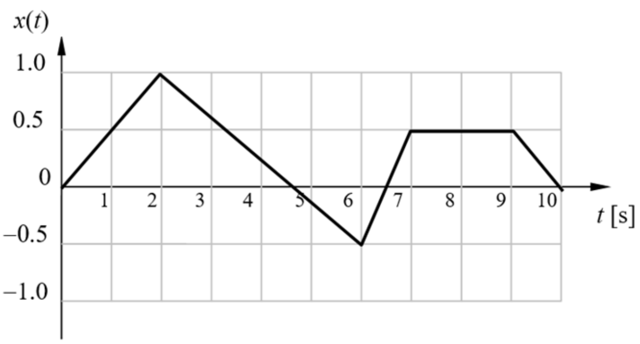
- Equivalent capacitance

(1.11-9) $C_p = C_1 + C_2 + \cdots + C_n$

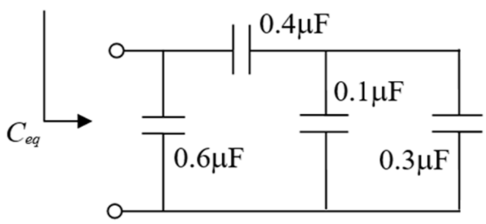
<u>Example</u>: If $C = 4.7\,\mu\text{F}$, then

(1) determine $i_C(t)$ for $x(t) = v_C(t)$ [V].

(2) determine $v_C(t)$ for $x(t) = i_C(t)$ [A].

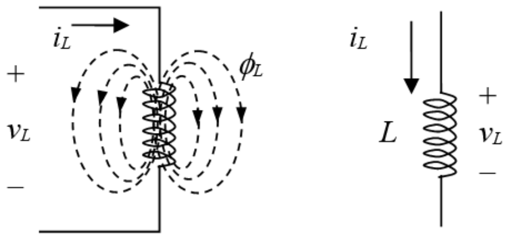


Example: Determine the equivalent capacitance C_{eq}.



1.12 Inductors

- Linear inductor



(1.12-1) $$v_L(t) = \frac{d\left(N\phi_L(t)\right)}{dt}$$

- Inductance

(1.12-2) $$L = \frac{N\phi_L(t)}{i_L(t)} \quad [\mathrm{H}]$$

N : the number of coils.

- Component equation (元件方程式)

(1.12-3) $$v_L(t) = L\frac{di_L(t)}{dt}$$

(1.12-4) $$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau$$

where $i_L(t_0) = \frac{1}{L}\int_{-\infty}^{t_0} v_L(\tau)\,d\tau$.

- Continuity of inductor's current

(1.12-5) $$i_L(t^-) = i_L(t) = i_L(t^+)$$

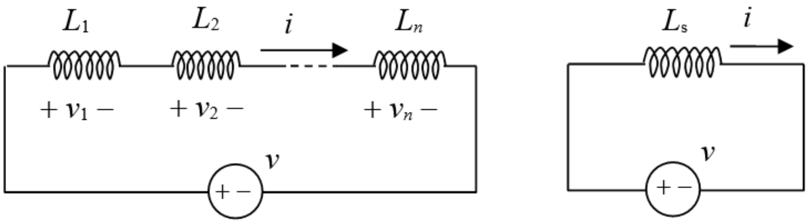
- Stored power and energy

(1.12-6) $$p_L(t) = v_L(t)i_L(t) = \left(L\frac{di_L(t)}{dt}\right)i_L(t) = \frac{d}{dt}\left(\frac{1}{2}Li_L^2(t)\right)$$

(1.12-7) $$w_L(t) = \int_{t_0}^{t} p_L(\tau)\,d\tau = \frac{1}{2}Li_L^2(\tau)\Big|_{i_L(t_0)}^{i_L(t)} = \frac{1}{2}Li_L^2(t) - \frac{1}{2}Li_L^2(t_0)$$

1.13 Inductors in Series and Parallel

1.13.1 Inductors in Series



- Mathematic model

(1.13.1-1) $v(t)=v_1(t)+v_2(t)+\cdots+v_n(t)=\sum_{k=1}^{n}v_k(t)$

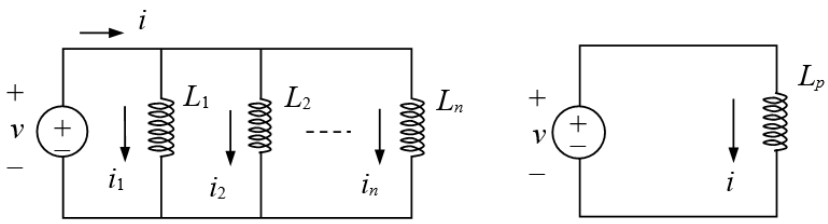
(1.13.1-2) $v_k(t)=L_k\dfrac{di(t)}{dt}$

KVL: $v(t)=\sum_{k=1}^{n}L_k\dfrac{di(t)}{dt}=(L_1+L_2+\cdots+L_n)\dfrac{di(t)}{dt}=L_s\dfrac{di(t)}{dt}$

- Equivalent inductance

(1.13-4) $L_s=L_1+L_2+\cdots+L_n$

1.13.2 Inductors in Series



- Mathematic model

(1.13.1-1) $i(t)=i_1(t)+i_2(t)+\cdots+i_n(t)=\sum_{k=1}^{n}i_k(t)$

(1.13.2-2) $i_k(t)=\dfrac{1}{L_k}\int_{-\infty}^{t}v(\tau)\,d\tau=i_k(t_0)+\dfrac{1}{L_k}\int_{t_0}^{t}v(\tau)\,d\tau$

KCL: $$i(t)=\sum_{k=1}^{n}\frac{1}{L_k}\int_{-\infty}^{t}v(\tau)\,d\tau$$

$$=\left(\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_n}\right)\int_{-\infty}^{t}v(\tau)\,d\tau=\frac{1}{L_p}\int_{-\infty}^{t}v(\tau)\,d\tau$$

- Equivalent inductance

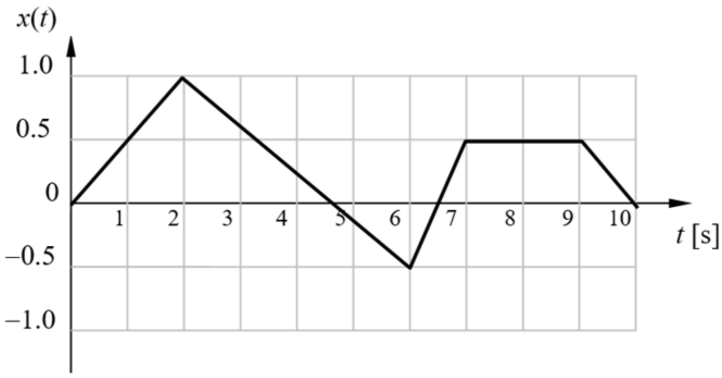
(1.13-8) $$\frac{1}{L_p}=\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_n}$$

(1.13-9) $$L_p=\left(\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_n}\right)^{-1}$$

<u>Example</u>: If $L=0.3$mH, then

(1) determine $i_L(t)$ for $x(t)=v_L(t)$ [V].

(2) determine $v_L(t)$ for $x(t)=i_L(t)$ [A].



Example: Determine the equivalent inductance L_{eq}.